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## **Why is scramble needed for DFE**

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# DFE Adaptation Algorithms: LMS and ZF

## ➤ Least Mean Squares(LMS)

- Heuristically arrive at optimal taps through traversal of the tap search space to the solution that minimizes the error metric

## ➤ Zero-Forcing(ZF)

- Adjust taps to directly zero out individual ISI components, as measured(usually) using pattern filtering to expose the ISI component of interest.

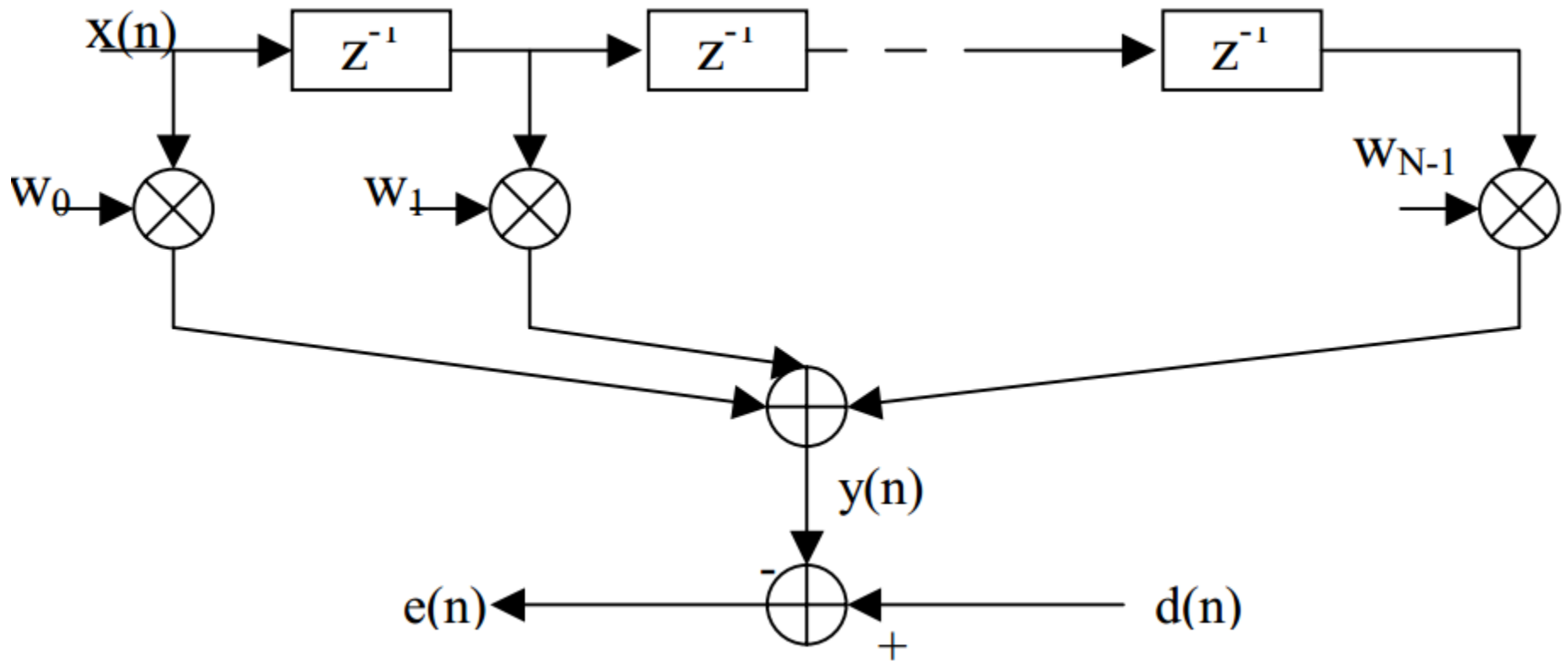
## ➤ LMS is the most popular algorithms

- The MMSE criterion ensures an optimum trade-off between residual ISI in  $d[k]$  and noise enhancement.
- MMSE equalizers achieve a significantly lower BEP compared to ZF equalizers at low-to-moderate SNRs.

# LMS Advantages

- **Easy to implement in software and hardware due to its computational simplicity and efficient use of memory.**
- **Robustly in the presence of numerical errors caused by finite-precision arithmetic**
- **Has been analytically characterized to the point where a user can easily set up the system to obtain adequate performance with only limited knowledge about the input and desired response signals.**

# Transversal Wiener Filter



# Performance Function

## ► MMSE Criterion

$$\begin{aligned}\xi &= E[e^2(n)] \\ &= E[(d^2(n)) - 2\bar{W}^T \bar{p} + \bar{W}^T R \bar{W}]\end{aligned}$$

$$\bar{w} = [w_0 \quad w_1 \quad \dots \quad w_{N-1}]^T$$

$$\bar{x}(n) = [x(n) \quad x(n-1) \quad \dots \quad x(n-N+1)]^T$$

$R = E[\bar{x}(n)\bar{x}^T(n)]$  ... autocorrelation matrix of the filter input

$\bar{p} = E[\bar{x}(n)d(n)]$  ... cross-correlation vector between  $\bar{x}(n)$   
and  $d(n)$

# Solution

## ➤ Wiener-Hopf Equation

$$R\bar{W}_0 = \bar{p}$$

## ➤ Steepest Descent

$$\nabla \xi = 2R\bar{W} - 2\bar{p}$$

$$\bar{W}(k+1) = \bar{W}(k) - \mu \nabla_k \xi$$

$$\bar{W}(k+1) = (1 - 2\mu R)(\bar{W}(k) - \bar{W}_0)$$

# LMS Solution

- LMS algorithm is a stochastic implementation of steepest descent algorithm. It simply replaces the cost function  $\xi = E[e^2(n)]$  by its instantaneous coarse estimate  $\hat{\xi} = e^2(n)$
- When the processes  $x(n)$  &  $d(n)$  are jointly stationary, this algorithm converges to a set of tap-weights which, on average, are equal to the Wiener-Hopf solution.
- The LMS algorithm is a practical scheme for realizing Wiener filters, without explicitly solving the Wiener-Hopf equation.

# LMS Equation

- Substituting cost function in the steepest descent recursion

$$\bar{W}(n+1) = \bar{W}(n) - \mu \nabla e^2(n)$$

$$\begin{aligned} \frac{\partial e^2(n)}{\partial w_i} &= 2e(n) \frac{\partial e(n)}{\partial w_i} \\ &= -2e(n) \frac{\partial y(n)}{\partial w_i} \\ &= -2e(n)x(n-i) \end{aligned}$$

- LMS equation

$$\bar{W}(n+1) = \bar{W}(n) + 2\mu e(n)\bar{x}(n)$$



# Coefficient Errors Vector

## ➤ Mean behavior of the coefficient errors vector

$$\mathbf{V}(n + 1) = \mathbf{W}(n) - \mathbf{W}_{opt}$$

$$\mathbf{V}(n + 1) = (\mathbf{I} - 2\mu\mathbf{R}_{xx})\mathbf{V}(n)$$

## ➤ Transformed coefficient error vector

$$\tilde{\mathbf{V}}(n + 1) = (1 - 2\mu\lambda_i)\tilde{\mathbf{V}}(n)$$

# Conditions for Convergence of the Mean coefficient Values

- The coefficient values only converge under:

$$\tilde{V}(n+1) = (1 - 2\mu\lambda_i)\tilde{V}(n)$$

$$-1 < (1 - \mu\lambda_i) < 1 \quad 0 < \mu < \frac{2}{\lambda_{max}}$$

$\lambda_{max}$  Is the maximum eigenvalue of  $\mathbf{R}_{xx}$

- Condition Number indicates the severity of the convergence problem.

$$\text{Condition Number: } \frac{\lambda_{max}}{\lambda_{min}}$$

- Highly-correlated input signals lead to large condition numbers and the LMS algorithm is slow to converge.

# Fourier Transform of Auto Correlation Matrix

## ➤ Fourier transform

$$S_{xx}(\omega) = \mathcal{F}\{r_{xx}(n)\}$$

## ➤ Relation to power spectrum

$$\lim_{L \rightarrow \infty} \left( \frac{\lambda_{max}}{\lambda_{min}} \right) = \frac{\max_{0 \leq \omega \leq \pi} S_{xx}(\omega)}{\min_{0 \leq \omega \leq \pi} S_{xx}(\omega)}$$

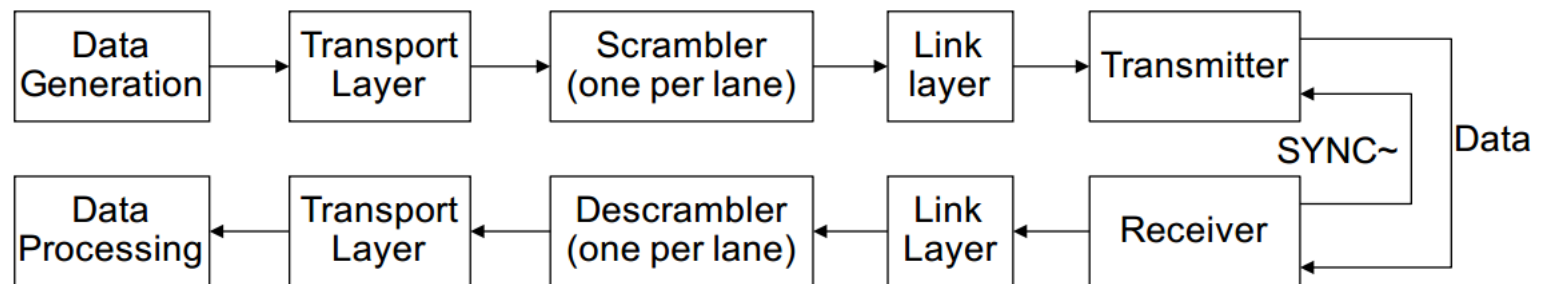
- The ratio of the largest and smallest values of the power spectral density of the input signal can be used to indicate the severity of the difficulty in achieving fast convergence of the LMS adaptive filter.
- The narrower the spectrum of the input signal, the slower the adaptive filter converges.

# Scramble Example---CPRI

Line bit rate	Scrambling support
614.4 Mbit/s	Not supported
1228.8 Mbit/s	Not supported
2457.6 Mbit/s	Not supported
3072.0 Mbit/s	Not supported
4915.2 Mbit/s	Recommended
6144.0 Mbit/s	Recommended
9830.4 Mbit/s	Recommended

# Scramble Example---JESD204

- Spectral peaks can cause electromagnetic compatibility or interference problems in sensitive applications. Via aliasing, they also cause code-dependent DC offsets in the data converters.
- Another advantage of scrambling is that it makes the spectrum data-independent, so that possible frequency-selective effects on the electrical interface will not cause data-dependent errors.



# Scramble Example---RapidIO

- Equalizers that can convert a single bit error into a multiple bit burst error, such as decision feedback equalizers (DFEs), shall not be used when IDLE1 has been selected for use on the link.
- IDLE2 was designed for LP-Serial links operating at greater than 5.5 GBaud and transmitters and receivers using adaptive equalization. In addition to the minimum idle sequence functionality, IDLE2 provides link width, lane identification and lane polarity information, randomized data for equalizer training and a command and status channel for receiver control of the transmit equalizer.

# Scramble Example--Ethernet

- All deploy scramble(64b/66b or 64b/67b) when linerate >3.125Gbps.

# Thanks

QA?